N1 Numbers

1) Thirty-three thousand, nine hundred and one.

2) 9741

N2 Ordering Numbers

1) Split the numbers into positive and negative:

$$-110, -1010, -1$$

and

$$11, 111, 1100$$

1100 is larger than 111, since it has more digits, and 111 is larger than 11 for the same reason.
-1010 is smaller than -110, since it is negative and has more digits, and -110 is smaller than -1 for the same reason.

So, we can order the numbers from smallest to largest, starting with the negative numbers:

$$-1010, -110, -1, 11, 111, 1100$$

2) Split the numbers into positive and negative:

$$-23, -28$$

and

$$54, 44, 37$$

Compare the first digits of the negative numbers: they are the same, so compare the second digits: 8 is bigger than 3, but since they are negative numbers, -28 is smaller than -23.
Compare the first digits of the positive numbers: 5 is bigger than 4 which is bigger than 3, so 54 is the biggest, then 44, then 37.

So, we can order the numbers from largest to smallest, starting with the positive numbers:

$$54, 44, 37, -23, -28$$

N3 Addition without a Calculator

Correct application of the column method will look like

Therefore, the solution to the problem is

$$1403 + 5519 = 6922$$

N4 Subtraction without a Calculator

Correct application of the column method will look like

Therefore, the solution to the problem is

$$3405 - 2567 = 838$$

N5 Addition and Subtraction Problems

1) Aaron has $23 + 45 + 6 = 74$ sweets

2) Jessie has $12 - 3 - 5 = 4$ marbles left

N6 Multiplication without a Calculator

a) To do the grid method, we construct a grid with 400, 30, and 3 across the top and 20 and 1 down the side. Then, we fill in the gaps in the grid by multiplying each component of 433 by each component of 21. This looks like:

×	400	30	3
20	8000	600	60
1	400	30	3

Now, we have to add the numbers inside the grid (shown in red) using whichever method you prefer. Adding along the rows, we get

$$8000 + 600 + 60 = 8660$$

$$400 + 30 + 3 = 433$$

Therefore, the final answer is

$$8660 + 433 = 9093$$

b) To do the grid method, we construct a grid with 500, 60, and 7 across the top and 100, 20, and 3 down the side. Then, we fill in the gaps in the grid by multiplying each component of 567 by each component of 123.

This looks like:

×	500	60	7
100	50000	6000	700
20	10000	1200	140
3	1500	180	21

Now, we have to add the numbers inside the grid (shown in red) using whichever method you prefer. Adding along the rows, we get

$$50000 + 6000 + 700 = 56700$$

$$10000 + 1200 + 140 = 11340$$

$$1500 + 180 + 21 = 1701$$

Therefore, the final answer is

$$56700 + 11340 + 1701 = 69741$$

N7 Division without a Calculator

a) To do the bus stop method, we first draw our "bus stop" with 768 on the inside and 8 on the outside. Then, we will ask "how many times does 8 go into each digit?", or if there is a remainder from the previous go, we form a new number by putting the remainder in front of the next digit, and we ask, "how many times does 8 go into this new number?" A correct application of the bus stop method will look like:

$$\begin{array}{r} 096 \\ 8\overline{)7^76^48} \end{array}$$

Therefore, the answer is 96

b) A correct application of the bus stop method will look like:

$$\begin{array}{r} 0126 \\ 9\overline{)1^11^23^54} \end{array}$$

Therefore, the answer is 126

N8 Multiplication and Division Problems

1) 65p = £0.65
So, the shop owner makes $150 \times £0.65 = £97.50$

2) $5 \div 0.4 = 12.5$ cups
However, there isn't enough juice to fill the 13th cup, so Sunita can only fill 12 cups.

N9 Checking Your Answers

1) $347 - 126 = 221$
Check: $221 + 126 = 347$

2) $356 \div 4 = 89$
Check: $4 \times 89 = 356$

N10 Order of Operations (BIDMAS)

There are two sets of brackets, so our first step should be to evaluate them both. Which one we do first doesn't matter, so here we'll choose the left one first. It only contains one operation, so we get

$$13 + 2 = 15$$

Now, the bracket on the right contains two operations: an index/power, and a division. We do the index first followed by the division:

$$36 \div 3^2 = 36 \div 9 = 4$$

Therefore, we only have 1 operation left in our calculation (the multiplication between the two brackets), so we get the answer to be

$$15 \times 4 = 60$$

N11 Fraction Basics

To get 20 on the bottom, we need to multiply the bottom by 4, because $5 \times 4 = 20$
The top needs to be multiplied by the same number, so

$$\frac{6}{5} = \frac{6 \times 4}{5 \times 4} = \frac{24}{20}$$

N12 More Fractions

1) Both 21 and 45 can be divided by 3, so

$$\frac{21}{45} = \frac{21 \div 3}{45 \div 3} = \frac{7}{15}$$

This cannot be simplified any further.

2)
$$495 \div 11 = 45$$

$$45 \times 3 = 135$$

N13 Adding and Subtracting Fractions

The bottom numbers need to be the same, so convert them all to equivalent fractions with 12 on the bottom, then solve:

$$\frac{5}{12} - \frac{1}{2} + \frac{3}{4} = \frac{5}{12} - \frac{1 \times 6}{2 \times 6} + \frac{3 \times 3}{4 \times 3}$$
$$= \frac{5}{12} - \frac{6}{12} + \frac{9}{12}$$
$$= \frac{5 - 6 + 9}{12}$$
$$= \frac{8}{12}$$
$$= \frac{2}{3}$$

N14 Mixed Numbers

$$29 \div 6 = 4 \text{ r } 5$$

The whole number part of the division is the number before the fraction, and the remainder goes on top with the dividing number on the bottom:

$$\frac{29}{6} = 4\frac{5}{6}$$

N15 Adding and Subtracting Mixed Numbers

Split up the whole number parts and fraction parts separately:

$$2\frac{7}{12} - 1\frac{1}{6} = 2 - 1 + \frac{7}{12} - \frac{1}{6}$$

Add/subtract each part:

$$2 - 1 = 1$$

and

$$\frac{7}{12} - \frac{1}{6} = \frac{7}{12} - \frac{2}{12} = \frac{5}{12}$$

Adding the parts together gives a final answer of:

$$1 + \frac{5}{12} = 1\frac{5}{12}$$

N16 Comparing Fractions

Firstly, find equivalent fractions, so that all fractions have 30 on the bottom:

$$\frac{13}{30}$$

$$\frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30}$$

$$\frac{1}{2} = \frac{1 \times 15}{2 \times 15} = \frac{15}{30}$$

$$\frac{8}{15} = \frac{8 \times 2}{15 \times 2} = \frac{16}{30}$$

Then, simply compare the top number of each fraction, and order them from smallest to largest:

$$\frac{12}{30} \quad \frac{13}{30} \quad \frac{15}{30} \quad \frac{16}{30}$$

Finally, replace these with the original fractions:

$$\frac{2}{5} \quad \frac{13}{30} \quad \frac{1}{2} \quad \frac{8}{15}$$

N17 Decimals

The first digit after the decimal point is the tenths. The second digit after the decimal point is the hundredths.
So, 1 is the number in the hundredths column in 7604.51382

N18 Adding and Subtracting Decimals

a) Add a 0 to the end of 2.86 so that both numbers have the same number of decimal places. Then, use the column method for addition, making sure to line up the decimal points:

$$
\begin{array}{r}
\overset{1}{}2\,.\,8\,6\,0 \\
0\,.\,2\,3\,1 \\
\hline
3\,.\,0\,9\,1
\end{array}
+
$$

So, $2.86 + 0.231 = 3.091$

b) Add a 0 to the end of 3.48 so that both numbers have the same number of decimal places. Then, use the column method for subtraction, making sure to line up the decimal points:

$$
\begin{array}{r}
\overset{0}{\cancel{1}}\,\overset{1}{1}\,.\,\overset{4}{\cancel{5}}\,\overset{1}{6}\,3 \\
3\,.\,4\,8\,0 \\
\hline
0\,8\,.\,0\,8\,3
\end{array}
-
$$

So, $11.563 - 3.48 = 8.083$

N19 Multiplying and Dividing Decimals

a) To divide decimals, we want to multiply them both by 10 repeatedly, until they're no longer decimals. In this case, if we multiply both numbers by 100, we can then work out

$$286 \div 20$$

And the answer will be the same as the answer to $2.86 \div 0.2$
Using the bus stop method here, we get

$$
\begin{array}{r}
0\,1\,4\,.\,3 \\
20\,\overline{)\,2\,{}^{2}8\,{}^{8}6\,.\,{}^{6}0\,0}
\end{array}
$$

Therefore, the solution is

$$2.86 \div 0.2 = 14.3$$

b) Convert the decimals to whole numbers, by moving the decimal point to the right, then we can calculate

$$433 \times 21$$

Using the grid method, we get:

×	400	30	3
20	8000	600	60
1	400	30	3

So, $433 \times 21 = 8000 + 600 + 60 + 400 + 30 + 3 = 9093$

We moved decimal points 2 places to the right in total initially, so we need to move the decimal point 2 places to the left to get our final answer:

$$43.3 \times 2.1 = 90.93$$

N20 Rounding and Estimating

1) 8 is bigger than 5, so 7.689 rounds up to 7.7

2) 2 is smaller than 5, so 134.25 rounds down to 134

N21 Percentages

1) Without a calculator:

10% of 160 = 16
5% of 160 = 8
1% of 160 = 1.6

$$38\% = 10\% + 10\% + 10\% + 5\% + 1\% + 1\% + 1\%$$
$$= 16 + 16 + 16 + 8 + 1.6 + 1.6 + 1.6$$
$$= 48 + 8 + 4.8$$
$$= 60.8$$

With a calculator:

38% of 160 = 0.38 × 160 = 60.8

2) 190 ÷ 250 = 0.76
0.76 × 100 = 76%

N22 Percentage Increase and Decrease

1) Multiplier = 4% + 100% = 104% = 1.04
Number of employees this year = 1.04 × 300 = 312

2) 1.5% of £40000 = £40000 × 0.015 = £600
Profit = £40000 − £600 = £39400

N23 Percentage Change

$$\text{Percentage change} = \frac{1800 - 2000}{2000} \times 100$$
$$= \frac{-200}{2000} \times 100$$
$$= -\frac{1}{10} \times 100$$
$$= -10\%$$

So, the population of the zoo has decreased by 10%.

N24 Reverse Percentages

Multiplier = 1 + 0.04 = 1.04

Let H be Tom's height from two years ago. We know that the result of multiplying H by 1.04 must be 182.

So, to find Tom's height two years ago, we need to divide 182 by 1.04

$$H = 182 \div 1.04 = 175 \text{ cm}$$

N25 Fractions, Decimals and Percentages

$$\frac{3}{4} = 0.75 = 75\%$$

N26 Converting Fractions, Decimals and Percentages

$$\frac{5}{8} = 5 \div 8 = 0.625$$

$$0.625 \times 100 = 62.5\%$$

N27 Comparing Fractions, Decimals and Percentages

Convert all to decimals:

$\frac{2}{25} = 2 \div 25 = 0.08$

8.5% ÷ 100 = 0.085

0.082

$\frac{9}{100} = 9 \div 100 = 0.09$

Now, we can order the decimals, from largest to smallest:

$$0.09, \ 0.085, \ 0.082, \ 0.08$$

Then, convert them back to their original form

$$\frac{9}{100}, \ 8.5\%, \ 0.082, \ \frac{2}{25}$$

N28 Ratio Basics

We know that Deborah's age is 28. Looking at the ratio, Deborah has 4 parts, meaning that 28 years constitutes 4 parts in the ratio. Therefore

$$1 \text{ part } = 28 \div 4 = 7$$

Kemah, Bob, and Deborah have 1, 2, and 4 parts in the ratio respectively. So

$$\text{Kemah's age } = 1 \times 7 = 7$$
$$\text{Bob's age } = 2 \times 7 = 14$$
$$\text{Deborah's age } = 28$$

N29 More Ratio

2 parts of almonds is 160 g, so

$$1 \text{ part} = 160 \div 2 = 80 \text{g}$$

Total number of parts $= 2 + 7 = 9$

$$\text{Total almond milk made} = 9 \times 80 = 720 \text{ g}$$

N30 Proportionality

This recipe makes 6 pancakes, but Wes wants to make 21.

$$\text{scale factor} = 21 \div 6 = 3.5$$

Therefore, he needs to 3.5 times as much of every ingredient. So, we get:

$$\text{flour: } 100 \times 3.5 = 350 \text{ g}$$

$$\text{eggs: } 2 \times 3.5 = 7 \text{ eggs}$$

$$\text{milk: } 300 \times 3.5 = 1050 \text{ ml}$$

N31 Direct and Inverse Proportion

30 Nurses = 2 hours
If we divide the left by 6 we need to multiple the right by 6. This gives the answer.

$$5 \text{ Nurses} = 12 \text{ hours}$$

N32 Formulas

$$F = 1.8C + 32$$
$$= 1.8 \times 35 + 32$$
$$= 63 + 32$$
$$= 95°F$$

M1 Units

millimetres (mm) and kilometres (km) are both metric units used to measure length.
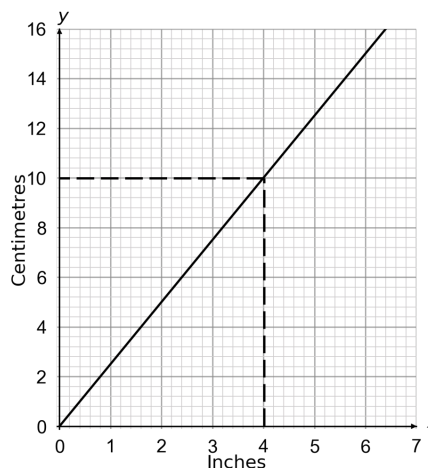
M2 Unit Conversions

1) 1 foot = 12 inches
So 72 inches = $72 \div 12 = 6$ feet
2) 3300 ml = $3300 \div 1000 = 3.3$ L

M3 Conversion Graphs

Go up from 4 on the x-axis until you meet the line.
Then go across until you meet the y-axis.

Read off the value.

4 inches = 10 centimetres

M4 Problems Involving Money

Set up cost = £30
Contract cost = £22.50 × 18 = £405
Total cost = £30 + £405 = £435

M5 Discounts and Increases

This is a percentage increase:

$$100\% + 20\% = 120\% = 1.2$$

The new price of the painting is

$$£75 \times 1.2 = £90$$

M6 Percentage Profit

To make a 45% profit, he needs to sell each bird box for

$$100\% + 45\% = 145\% = 1.45$$

times the amount it costs him to make one.
So, he needs to sell them for

$$£12 \times 1.45 = £17.40$$

M7 Best Buys

4-packs:
He would need to buy $8 \div 4 = 2$ packs of 4.
This would cost £1.50 × 2 = £3.00
However, there is 20% off, so they would cost £3.00 × 0.8 = £2.40

Individually: Individual bars are 'buy one get one free' so he will get 2 for 65p.
He needs to buy $8 \div 2 = 4$ lots of 2 bars.
This would cost £65p × 4 = 260p = £2.60

Therefore, he should buy the four-pack of chocolate bars to get the best deal.

M8 Interest and Compound Interest

After 1 year: £800 × 1.02 = £816
After 2 years: £816 × 1.02 = £832.32
After 3 years: £832.32 × 1.02 = £848.9664 = £848.97 (nearest penny)

So, Mick will have £848.97 in his savings account after 3 years.

M9 Money in the Real World

20% is added on, so multiply the original value by 1.20
Value of t-shirt after VAT added = £32.50 × 1.20 = £39

M10 Speed

15 minutes = 15 ÷ 60 = 0.25 hours

$$\text{speed} = \text{distance} \div \text{time} = 5 \div 0.25 = 20 \text{ mph}$$

M11 Density

$$\text{density} = \text{weight} \div \text{volume} = 1.5 \div 5 = 0.3 \text{ g/cm}^3$$

M12 Perimeters of Shapes

1) A square has all sides the same length, so

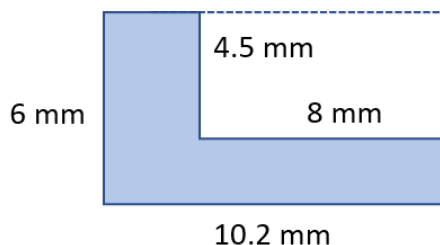$$\text{perimeter} = 9 + 9 + 9 + 9 = 36 \text{ m}$$

2) An isosceles triangle has two sides the same length. The sides which are the same length are 7 cm.

$$\text{perimeter} = 7 + 7 + 3 = 17 \text{ cm}$$

M13 Areas of Shapes

There is a large rectangle with sides 6 mm and 10.2 mm, and a smaller rectangle with sides of 4.5 mm and 8 mm has been cut out of this.

Area of large rectangle = 6 × 10.2 = 61.2 mm^2
Area of small rectangle = 8 × 4.5 = 36 mm^2

Area of shape = 61.2 − 36 = 25.2 mm^2

M14 Circles and Pi

The diameter of a circle is twice the radius of the circle.

M15 Area and Circumference

Area of semicircle = $\frac{1}{2}$ area of circle = $\frac{1}{2} \times \pi \times 5^2 = 39.3$ m^2 (1 dp)

Perimeter of semicircle = $\frac{1}{2}$ circumference of circle + diameter

$\frac{1}{2}$ circumference of circle = $\frac{1}{2} \times 2\pi \times r = \pi \times r = \pi \times 5 = 15.70796...$ m

diameter = $2 \times r = 2 \times 5 = 10$ m

Perimeter = 15.70796... + 10 = 25.70896... = 25.7 m (1 dp)

M16 Using Length and Area in Calculations

Convert all lengths to cm:
width of floor = 2.4 × 100 = 240 cm
length of floor = 3.3 × 100 = 330 cm

Area of floor = 240 × 330 = 79200 cm^2

Area of each tile = 30 × 30 = 900 cm^2

Number of tiles needed = 79200 ÷ 900 = 88

M17 3D Shapes

The container is 15 cm by 6 cm by 3 cm
or
15 cm × 6 cm × 3 cm
or
15 cm long, 6 cm wide and 3 cm high

M18 Volume of 3D Shapes

A cube has all sides the same length. In this case, the length, width and height are all 9.2 mm.

$$\text{Volume} = 9.2 \times 9.2 \times 9.2 = 778.688 \text{ mm}^3$$

M19 Volume of Prisms and Cylinders

Calculate the area of the cross-section using the formula for the area of a circle: $A = \pi r^2$
So in this case, the cross-sectional area is:

$$\text{Area} = \pi \times 3^2 = 3.14 \times 9 = 28.26 \text{ cm}^2$$

Multiply this by the length to get the volume:

$$\text{Volume} = 28.26 \times 8 = 226 \text{ cm}^3 \text{ (nearest whole number)}$$

M20 Nets of 3D Shapes

This is a net of a triangular prism - there are two triangles which are the front and back and three rectangles which are the long sides.

M21 Surface Area of 3D Shapes

Area of front face $= \dfrac{1}{2} \times 10 \times 12 = 60$ m^2

The front and back faces are the same, so

Area of back face $= 60$ m^2

Area of base $= 10 \times 22.4 = 224$ m^2
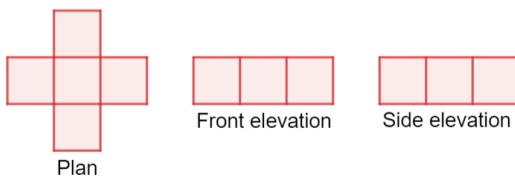
Area of side face $= 13 \times 22.4 = 291.2$ m^2

The two side faces are the same, so

Surface area $= 60 + 60 + 224 + 291.2 + 291.2 = 926.4$ m^2

M22 Plans and Elevations

The shape is in the shape of a cross, so the plan is going to be the shape of a cross. The front and side elevations are the same - both are 3 blocks wide and 1 block tall. All 3 projections are shown below.

Plan   Front elevation   Side elevation

M23 Maps and Scale Drawings

The length of the ladybird in the drawing is 2.6 cm.

The ratio of length in drawing to length in real life is 1 : 0.2, so the measurements in real life must be 0.2 times the size in the drawing.

So, length in real life $= 2.6 \times 0.2 = 0.52$ cm

M24 Making Scale Drawings

Convert all measurements to cm:

height $= 0.75 \times 100 = 75$ cm

width $= 1.2 \times 100 = 120$ cm
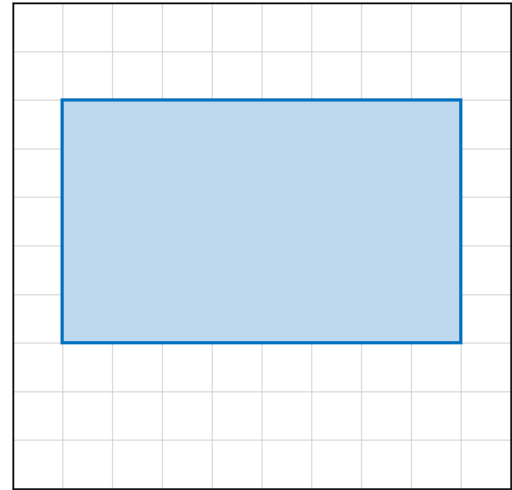
The ratio of drawing to real life is 1 : 15, so the measurements in the drawing must be 15 times smaller than those in real life.

So,

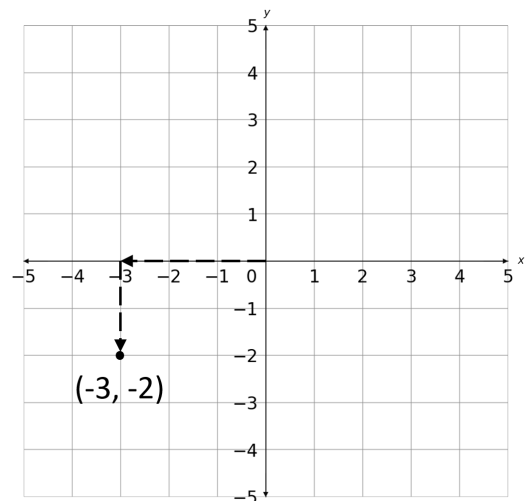height in drawing $= 75 \div 15 = 5$ cm

width in drawing $= 120 \div 15 = 8$ cm

The drawing will look like the following.

M25 Coordinates

Go across to -3 on the x-axis, and then down to -2 on the y-axis. Plot the coordinate using a dot or a cross:

(-3, -2)

M26 Angles in 2D Shapes

Angles in a triangle add up to 180°.
So,

$$x = 180 - 70 - 50 = 60°$$

D1 Median and Mode

a) To find the median, we must first put the values in ascending order:

167, 169, 175, 175, 175, 176, 181, 182, 184, 210

There are 10 numbers, so the median is half way between the middle two numbers: 175 and 176.
Therefore, the median is 175.5 cm (the halfway point).

b) The mode is the most common number in the list, which is 175

D2　Mean and Range

a) There are 10 numbers in the list, so we must add up all the values and divide by 10.

$$\text{mean} = \frac{181 + 182 + 175 + 176 + 210 + 169 + 175 + 184 + 167 + 175}{10}$$

$$= 179.4 \text{ cm}$$

b) The range is the difference between the largest value in the list and the smallest value in the list.

$$\text{range} = \text{biggest value} - \text{smallest value} = 210 - 167 = 43 \text{ cm}$$

D3　Comparing Data Sets

To compare two similar sets of data, we must calculate the range of each set and compare them. The data set with the smallest range is the most consistent.
The range is the difference between the largest value in the list and the smallest value in the list.

$$\text{range of set A} = \text{biggest value} - \text{smallest value} = 210 - 167 = 43$$

$$\text{range of set B} = \text{biggest value} - \text{smallest value} = 182 - 162 = 20$$

The range is smallest in set B, therefore the height of the men is more consistent in set B.

D4　Estimating the Mean

Calculate the midpoints of each group, and write them in a midpoint column.
Then, calculate the 'frequency × midpoint' for each group.
Then, calculate the total frequency and total 'frequency × midpoint'

Age	Frequency	Midpoint	Frequency × midpoint
18-30	14	24	336
31-35	4	33	132
36-50	12	43	516
51-75	9	63	567
Total:	39		1551

Finally, estimate the mean:

$$\text{Mean} = (\text{Total 'frequency'} \times \text{midpoint'}) \div (\text{Total frequency})$$

$$= 1551 \div 39 = 39.8 \text{ (1 dp)}$$

D5　Probability Basics

a) the probability of rolling any number on a fair six sided die is the same. So, Paul is equally likely to roll a 6 or a 1.

b) There are 3 odd numbers on a six sided die: 1, 3 and 5.
So, the probability of rolling an odd number is $\frac{3}{6} = \frac{1}{2} = 0.5$

c) There is no number 10 on the six sided, so the probability of rolling a 10 is 0.

D6　More Probability

a) There are 6 numbers on the six sided die, so the probability of landing on the number 1 is $\frac{1}{6}$

Therefore, the probability of **not** landing on the number 1 is $1 - \frac{1}{6} = \frac{5}{6}$

b) We need to multiply the probability that the number will not land on the number 1 by itself.

$$\text{probability} = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

D7　Two-Way Tables

a)

	Standard ticket	VIP ticket	Total
Adult	32	15	47
Child	12	3	15
Total	44	18	62

b) 18 out of 62 people purchased a VIP ticket. So the probability is

$$\frac{18}{62} = \frac{9}{31}$$

D8　Tables for Multiple Events

Draw a table for multiple events and fill this in:

		Die One					
	+	1	2	3	4	5	6
Die Two	2	3	4	5	6	7	8
	4	5	6	7	8	9	10
	6	7	8	9	10	11	12
	8	9	10	11	12	13	14
	10	11	12	13	14	15	16
	12	13	14	15	16	17	18

There are 36 numbers in total. 3 of the numbers are 10. Therefore the probability is

$$\frac{3}{36} = \frac{1}{12}$$

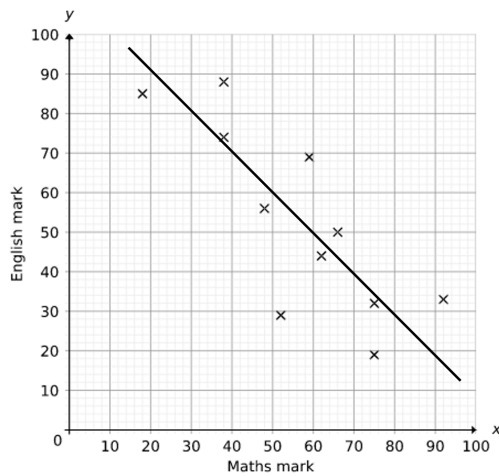### D9　Scatter Graphs and Correlation

a) As the x value increases, so does the y value. Therefore, this graph displays positive correlation.

b) There appears to be no relationship followed by the points on this graph. Therefore, it displays no correlation.
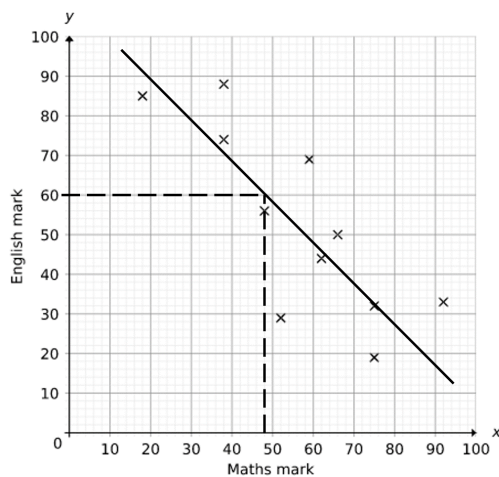
c) As the x value increases, the y value decreases. Therefore, this graph displays negative correlation.

D10 Line of Best Fit

a) Draw a line that goes through the middle of all the points, with roughly an equal number of points on either side of the line.

b) Go across from 60 on the y-axis until you meet the line, then go down until you meet the x-axis. Read off the value.

You would predict that someone who achieved 60 on their English exam would score 50 on their maths exam.